Name: Date: Period: 1

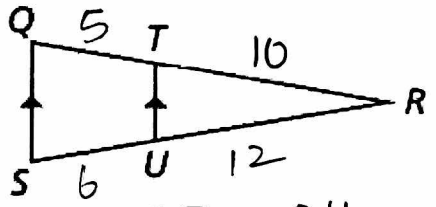
Proportionality Theorems

Lesson Objective IWBAT apply proportionality thms in Δs.

Triangle Proportionality Theorem ★★

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

|| segments → sides proportional
p → q

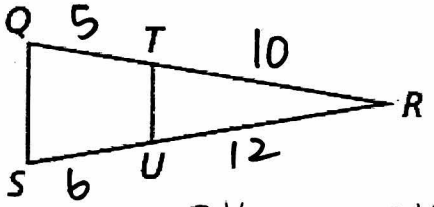


$\overline{QS} \parallel \overline{TU} \rightarrow \frac{RT}{QT} = \frac{RU}{SU}$

Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

sides proportional → || segments.



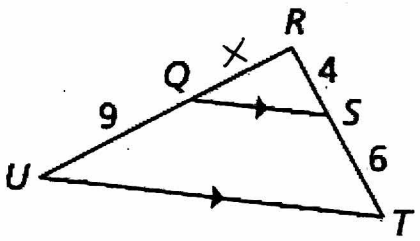
$\frac{RT}{QT} = \frac{RU}{SU} \rightarrow \overline{QS} \parallel \overline{TU}$

Example:

1. In the diagram, $\overline{QS} \parallel \overline{UT}$, RS = 4, ST = 6, and QT = 9. What is RQ?

$\frac{x}{9} = \frac{4}{6} \rightarrow 6x = 36 \rightarrow x = 6$

RQ = 6



BRIEF REVIEW: Which statements, formed by an original conditional statement, are logically equivalent?

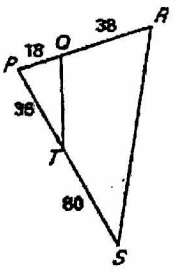
- The conditional and the contrapositive (p → q; ~q → ~p)
- The converse and the inverse (q → p; ~p → ~q)

The theorems in the solid boxes above imply the following:

Contrapositive of the Triangle Proportionality Theorem	Inverse of the Triangle Proportionality Theorem
sides not proportional → segments not ||	segments not || → sides not proportional

Example:

2. Determine whether $\overline{QT} \parallel \overline{RS}$.



$\frac{18}{38} \stackrel{?}{=} \frac{36}{80}$

$\frac{9}{19} \neq \frac{9}{20}$

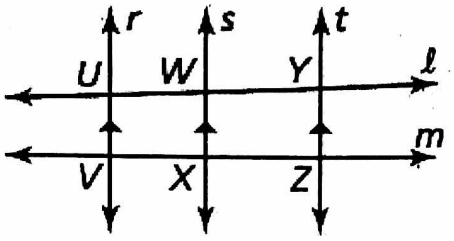
NOT ||

*cross-products

1440 ≠ 1368

Three Parallel Lines Theorem ★Perf. Task★

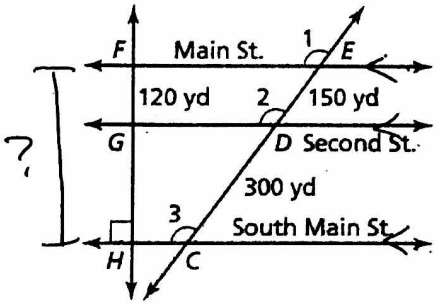
If three parallel lines intersect two transversals, then they divide the transversals proportionally.



$r \| s \| t \rightarrow \frac{UW}{WY} = \frac{VX}{XZ}$

Example:

3. In the diagram, $\angle 1 \cong \angle 2 \cong \angle 3$. GF = 120 yd, DE = 150 yd, and CD = 300 yd. Find the distance HF between Main Street and South Main Street.



$\frac{120}{x} = \frac{150}{300}$ 1 2

$x = 240$

$HF = 360$

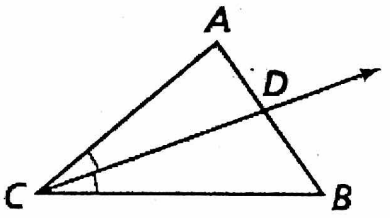
$\frac{120}{y} = \frac{150}{450}$ 1 3

$y = 360$

$HF = 360$

Triangle Angle Bisector Theorem

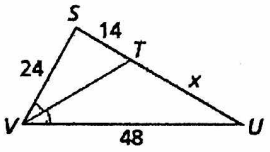
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



$\frac{AD}{DB} = \frac{AC}{CB}$

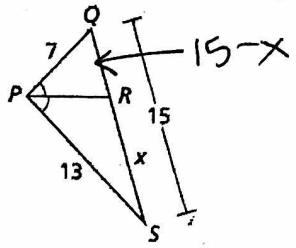
Example:

4. Find the value of x.



$\frac{14}{x} = \frac{24}{48}$ 1 2 $\rightarrow$ $x = 28$

5. Use the given side lengths to find RS.



$\frac{15-x}{x} = \frac{7}{13}$

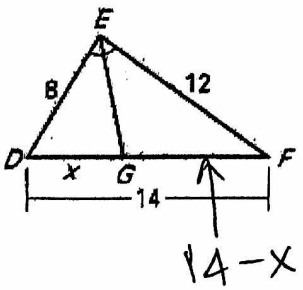
$7x = 195 - 13x$

$20x = 195$

$x = 9.75$

$RS = 9.75$

6. Use the given side lengths to find FG.



$\frac{14-x}{x} = \frac{12}{8}$ 3 2

$3x = 28 - 2x$

$5x = 28$

$x = 5.6$

$FG = 8.4$